REPORT DOCUMENTATION PAGE

Form Approved
OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave Blank)

2. REPORT DATE
1/8/03

3. REPORT TYPE AND DATES COVERED
Final Report
April 1, 1998 - March 31, 2002

4. TITLE AND SUBTITLE
Shape and Surface Reconstruction, Quantification and Deformation

5. FUNDING NUMBERS
ARO # DAAG55-98-1-0177

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Illinois, Champaign, Illinois | Duke University, Durham, North Carolina

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
37578.12-MA

11. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12 a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12 b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

Research under this grant focused on the reconstruction and the representation of continuous shapes and objects. The primary data structure are meshes, and we study triangle meshes for surface and tetrahedron meshes for volume data. Beyond these two types, we obtained new results for constructing quadrangular meshes for surfaces. Our approach to these problems was fairly broad, ranging from problem modeling, to algorithm design, to implementing and experimenting, and to application finding and adaptation.

14. SUBJECT TERMS

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION ON THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev.2-89)
Prescribed by ANSI Std. 239-18
298-102

Enclosure 1

20030403 003

Final Report for grant DAAG55-98-1-0177

The research under this grant focused on the reconstruction of physical objects In this report we distinguish between (1) research on geometric-topological foundations and algorithm development, (2) research related to technology transfer and targeting the problem definition, (3) implementation work and computer experimentation. This division corresponds roughly but not precisely to the responsibilities of the supported personnel. The grant supported the work of Herbert Edelsbrunner, Arts and Sciences Professor of Computer Science and Mathematics at Duke University. His role was the theoretical formulation and solution of problems. He has expertise in algorithms, geometry, and topology. The grant payed for a small fraction of the time of Ping Fu, for her contributions in technology transfer. Ping Fu is also CEO and President of Raindrop Geomagic and she is a Research Associate Professor in the Computer Science Department of Duke University. Her work includes the formulation of theoretical problems motivated by practice as well as the identification of industrial applications for new theoretical results. The grant payed for a 50% Research Assistantship, which was used to hire Ho-Lun (Alan) Cheng and Afra Zomorodian, both graduate students at the University of Illinois at Urbana-Champaign, the former institute of the PI. The papers submitted with this report are

[1] H. Edelsbrunner, M. A. Facello, P. Fu, J. Qian, D. V. Nekhayev. Wrapping 3D scanning data. *In* "Proc. IS&N/SPIE's Sympos. Electronic Imaging, 1998", San Jose, California, 148–158.

[2] T. K. Dey, H. Edelsbrunner, S. Guha and D. V. Nekhayev. Topology preserving edge contraction. *Publ. Inst. Math. (Beograd) (N. S.)* **66** (1999), 23–45.

[3] A. Zomorodian and H. Edelsbrunner. Fast software for box intersections. *Internat. J. Comput. Geom. Appl.* **12** (2002), 143–172.

[4] H.-L. Cheng, H. Edelsbrunner and P. Fu. Shape space from deformation. *Comput. Geom. Theory Appl.* **19** (2001), 191–204.

[5] H.-L. Cheng, T. K. Dey, H. Edelsbrunner and J. Sullivan. Dynamic skin triangulation. *Discrete Comput. Geom.* **25** (2001), 525–568.

[6] H. Edelsbrunner, D. Letscher and A. Zomorodian. Topological persistence and simplification. *Discrete Comput. Geom.* **28** (2002), 511–533.

[7] H. Edelsbrunner, J. Harer and A. Zomorodian. Hierarchical Morse-Smale complexes for piecewise linear 2-manifolds. *Discrete Comput. Geom.*, to appear.

[8] H. Edelsbrunner and R. Waupotitsch. Adaptive simplicial grids from cross-sections of monotone complexes. *Internat. J. Comput. Geom. Appl.* **10** (2000), 267–284.

[9] H. Edelsbrunner and D. Guoy. Sink-insertion for mesh improvement. *Internat. J. Found. Comput. Sci.* **13** (2002), 223–242.

[10] H. Edelsbrunner and D. R. Grayson. Edgewise subdivision of a simplex. *Discrete Comput. Geom.* **24** (2000), 707–719.

[11] S.-W. Cheng, T. K. Dey, H. Edelsbrunner, M. A. Facello, and S.-H. Teng. Sliver exudation. *J. Assoc. Comput. Mach.* **47** (2000), 883–904.

[12] H. Edelsbrunner and D. Guoy. An experimental study of sliver exudation. *Engineering w. Computers* **18** (2002), 229–240.

We structure the report in four sections corresponding to the main four topics covered by the twelve publications: surface reconstruction and simplification ([1, 2, 3]), surface deformation and shape space ([4, 5]), topological features ([6, 7]), and volume meshing ([8, 9, 10, 11, 12]).

I. SURFACE RECONSTRUCTION AND SIMPLIFICATION

Paper [1] describes the capabilities of our surface reconstruction algorithm. Given a set of points in three-dimensional space, this algorithm reconstructs the surface from which the points are sampled. Depending on the data, the surface may be ambiguous or may not exist at all. Any algorithm has to have a way to deal with such situations. The paper illustrates the strengths and weaknesses of our algorithm by showing how it performs for a variety of input data. In many ways, the surface reconstruction problem is at the core of our work on surfaces and at least partially motivates the other research conducted. Ping Fu played a key role in identifying problems that needed solutions to improve or extend out capability to reconstruct surfaces. She also played a key role in prioritizing these problems.

An extension of surface reconstruction is surface simplification. It is frequently the case that the input data is over-sampled, which leads to heavy surface representations consisting of too many triangles. To *simplify* a surface means to find a coarser triangulation that represents the same shape, up to some accuracy. We implemented the edge contraction algorithm described in M. GARLAND AND P. S. HECKBERT, Surface simplification using quadratic error metrics. *Comput. Graphics*, Proc. SIGGRAPH 1997, 209–216. A sub-problem that has not been solved in that paper is how to prevent the algorithm to change the topological type of the surface. Paper [2] gives a complete analysis of this problem, both for two- and for three-dimensional triangulations. The two-dimensional results have been fully implemented in our software.

Paper [3] studies the implementation of a fast algorithm for finding intersecting boxes. The motivation for this work was the isolation of self-intersection detection as a subproblem of the surface reconstruction problem. Given a triangulated surface, we enclose each triangle by a box and test triangle pairs for intersections if their bounding boxes overlap. Later we found additional applications, such as determining the distance between two surfaces or a surface and a point set. The theoretical algorithm used in our implementation is based on the idea of streaming a segment tree, as described in H. EDELSBRUNNER AND M. H. OVERMARS, Batched dynamic solutions to decomposable searching problems, *J. Algorithms* **6** (1985), 515–542. Besides giving the implementation, the main result in [3] is the documentation of the dramatic effect of hybridization as an algorithmic technique that can lead to substantial improvements in the running time.

II. SURFACE DEFORMATION AND SHAPE SPACE

Our work in this area is based on the class of surfaces introduced in H. EDELSBRUNNER, Deformable smooth surface design, *Discrete Comput. Geom.* **21** (1999), 87–115. Surfaces in this class are referred to as *skins*; they are piecewise quadratic and have continuous normal directions and maximum curvatures. Each skin surface is specified by a collection of weighted points in three-dimensional space and uses advanced geometric concepts based on the Voronoi diagram to achieve computational efficiency.

Paper [4] exploits the fact that skin surfaces deform continuously when we vary the controlling points and their weights. With this property, it is possible to define canonical deformations of one shape to another or, more generally, between $k+1 \geq 2$ shapes. The deformation operation forms a k-dimensional vector space of shapes. This space is a powerful structure with applications in computer graphics (where the idea of morphing between more than two shapes has been picked up and has recently flourished), in drug design (where one is concerned with the variety of the molecular shapes with medical applications), and other areas in which geometric shapes play a significant role.

Paper [5] describes an algorithm that maintains the mesh of a deforming surface. It combines operations that adapt the surface to changing shape (which amounts to moving the mesh vertices), to changing curvature (which amounts to changing the local density of vertices), and to changing topology (which requires local re-connections within the mesh.) The implementation of the algorithm is delicate and is still being improved. The most common current use of the software is for constructing smooth surfaces of protein conformations. The high quality of the mesh supports down-stream applications, including the 3D printing of these molecules using layered technology and the computation of electro-static distributions using the finite element method.

III. Topological features

The two topological concepts studied here are motivated directly by work described above.

The motivation for paper [6] is the observation that noise in the data cannot be treated with geometric methods alone but requires quantifiable topological methods as well. We approach this problem by introducing the topological persistence of features in a geometric or topological object given as a filtration. The *filtration* is a nested sequence of complexes, and each complex represents the view of the object at some resolution. Intuitively, the *persistence* of a feature is the width of the resolution window in which the feature appears as part of the shape description. In other words, it is a numerical expression of how persistent the feature is under changes of the resolution. The main result of the paper is that this intuitive notion can be made concrete in terms of the homology groups of the complexes in the filtration. The paper also gives an algorithm for computing the persistence and presents experimental results obtained with its implementation.

Part of the motivation for paper [7] is the need to decompose potentially complicated surfaces into patches that, in some sense, are natural and conform with what might call the topological features of the surface. We attach the problem using the concept of a Morse-Smale complex for a piecewise linear height function. Our main contribution is the development of algorithmic techniques that permit the construction of such complexes through the simulation of a smooth height function. The key point here is that a Morse-Smale complex is really only defined from smooth functions, while data in practice is sampled and after interpolation only piecewise linear continuous. The notion of topological persistence discussed above is then used to define and construct a hierarchy of progressively simpler Morse-Smale complexes. This hierarchy is essential if we want to apply Morse-Smale complexes to continuous problems, for which all useful notions of features are scale dependent. Typically, the appropriate scale level depends on the context and cannot be predetermined.

IV. Volume meshing

In this area we consider tetrahedral representations of three-dimensional domains. The first three papers study the problem of mesh refinement from different angles. The last two papers describes a solution to the problem of slivers in three-dimensional Delaunay tetrahedrizations.

Paper [8] extends work on the incremental construction of Delaunay triangulations, reported for example in H. Edelsbrunner and N. R. Shah, Incremental topological flipping works for regular triangulations, *Algorithmica* **15** (1996), 223–241. It does this by forming hierarchies that can be used for on-line and local mesh-refinement and -coarsening. Paper [9] suggests that certain circumcenters of tetrahedra are better suited for mesh-refinement than others. This claim is substantiated with a suite of experiments that show that the restriction to these special circumcenters leads to numerical improvements and computational savings. Paper [10] extends the common subdivision of a triangle into four similar triangles to tetrahedra and higher-dimensional simplices. We expect that this extension will be used to generalize the popular subdivision method in computer graphics to three- and higher-dimensional domains.

A *sliver* in a three-dimensional Delaunay triangulation is a relatively flat tetrahedron. Its combination of small and large angles creates troubles down-stream for numerical algorithms using the triangulation as a mesh. Slivers have been recognized as a serious stumbling block in scientific computations, but progress on methods that can eliminate slivers has been slow. Paper [11] is considered a breakthrough in this area. It shows that slivers can be eliminated by assigning modest weights to the points and change the Delaunay into a weighted Delaunay triangulation. The algorithm for eliminating slivers is fast, but the numerical bound on non-flatness we managed to proof has been disappointingly small. Paper [12] shows that the numerical bound that can be achieved in practice is about 5°, which is significantly larger than the pessimistic theoretical bound.